1. [*4 marks*]

(a) It is a predicate; the free variable is n.

(b) It is a statement.

(c) It is a statement; the predicate has been quantified.

(d) It is not a statement or predicate.

2. [*6 marks*]

(a) $E(n) \implies O(n^2+1)$, where $E(x) =$"x is an even number" and $O(x) =$"x is an odd number".

(b) $P(11,111,111)$, where $P(x) =$"x is a prime number".

(c) $(\forall x \in \mathbb{Z})(M(x,12) \implies M(x,4))$, where $M(x,y) =$"x is a multiple of y".

3. [*2 marks*] The first statement tells us that there is some antidote for every poison. The second statement also tells us this, and also asserts that it is the **same** chemical which acts as the antidote for every poison.

If the second statement is true, then the first statement is certainly true; however, the first statement does not imply the second.

4. [*8 marks*]

(a)

A	B	$(A \wedge B)$	$\implies$	$(A \vee B)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	F

(b)

A	B	$(A \implies B)$	$\implies$	$\neg A$
T	T	T	F	F
T	F	F	T	F
F	T	T	T	T
F	F	T	T	T

(c)

A	B	$(A \implies B)$	$\implies$	$(\neg B$	$\implies$	$\neg A)$
T	T	T	T	F	T	F
T	F	F	T	T	F	F
F	T	T	T	F	T	T
F	F	T	T	T	T	T

(d)

A	B	**$A \wedge$**	$((A \implies B)$	$\wedge$	$(A \implies$	$\neg B))$
T	T	F	T	F	F	F
T	F	F	F	T	T	T
F	T	F	T	T	T	F
F	F	F	T	T	T	T

5. [*2 marks*] The statements (a) and (c) are tautologies.

6. [*2 marks*] The statement (d) is the only contradiction. If $A =$"It is Friday" and $B =$"We will watch TV", then the statement translates to: "It is Friday, and if it is Friday we will watch TV, and if it is Friday we will not watch TV".

This statement cannot be true, since it can only be true if it is Friday (A is true).

But if A is true, then for both of the implications to be true, B must be true, and $\neg B$ must be true; so B is both true and false (we will watch TV and we will not watch TV), which is not possible. So the statement cannot be true.

7. [*16 marks*]

(a) $B(m,n) =$"If n is not a factor of m, then n is not a factor of m^2."

(b) $C(m,n) =$"It is not true that, if n is a factor of m^2, then n is a factor of m."

(c) $D(m,n) =$"If n is factor of m, then n is a factor of m^2."

(d) $A(1,n)$ is TRUE for any value of n. That is, if n is factor of 1, then n is a factor of 1. This is true because it is of the form $P(n) \implies P(n)$, which is true regardless of the meaning of $P(n)$.

Since the contrapositive is true for exactly the same cases as $A(m,n)$, when $m = 1$, $B(1,n)$ is also TRUE for any value of n.

Let $m = 1$. Again, since $1^2 = 1$, $D(1,n)$ is of the form $P(n) \implies P(n)$, so $D(1,n)$ is TRUE.

(e) $(\forall m)(\forall n)A(m,n)$ is not true. For example, 4 is a factor of 36, but 4 is not a factor of 6. So $(m,n) = (4,6)$ is a counterexample. It is FALSE.

The negation is $\neg(\forall m)(\forall n)A(m,n)$. That is, $(\exists m)(\exists n)\neg A(m,n)$; "there are some values of m and n which make $A(m,n)$ true. We have already shown this is true, for $m = 1$, and any value of n. So the statement is TRUE.

The converse $D(m,n)$ is TRUE. Suppose n is a factor of m. Then $m = nk$ for some integer k. Now consider m^2. Since n is a factor of m, we have $m^2 = (nk)^2 = n \cdot nk^2$ (where $nk^2 \in \mathbb{Z}$), so n is a factor of m^2, as required.

TOTAL MARKS: 40