NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

PLEASE SHOW ALL WORKING.

1. (**10 marks**) Given that $*$ is a group operation on the set $G = \{a, b, c, d\}$.

(a) (**6 marks**) Complete the following Cayley table:

$*$	a	b	c	d
a	c			
b		c		
c			c	
d				

(b) (**4 marks**) Find the identity e_G of G, and find the inverses b^{-1} and d^{-1} in G.

2. (**8 marks**) Let $\mathrm{GL}_2(\mathbb{R})$ be the set of all invertible 2×2 matrices over $\mathbb{R}$. As shown in the class $\mathrm{GL}_2(\mathbb{R})$ is a group under matrix multiplication. Let H be a subset of $\mathrm{GL}_2(\mathbb{R})$ such that

$$H = \{A \in \mathrm{GL}_2(\mathbb{R}) : \det(A) = 1\}.$$

Show that H is a subgroup of $\mathrm{GL}_2(\mathbb{R})$.

3. (**10 marks**)

(a) (**4 marks**) Construct a Cayley table of $(\mathbb{Z}_4, +_4)$.

(b) (**6 marks**) Show that the group $\mathbb{Z}_4$ is isomorphic to the group $G = \{1, i, -1, -i\}$, where $i = \sqrt{-1}$ and G is the group under the complex multiplication.

4. (**12 marks**) Let $S_3 = \{e = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}, \varphi = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}, \psi = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}, \alpha = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}, \beta = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}, \gamma = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{bmatrix}\}$ be the symmetric group, and $K = \{e, \alpha\}$ a subset of the symmetric group S_3.

(a) (**4 marks**) Show K is a subgroup of S_3.

(b) (**8 marks**) Find all left cosets xK of K in S_3.