NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

PLEASE SHOW ALL WORKING.

1. (**5 marks**) Use congruence to show that for every $n \in \mathbb{N}$, $9(n+12)(11n+25)$ is divisible by 6.

2. (**15 marks**)

(a) (**5 marks**) Let $f(x)$ and $g(x)$ be polynomials of $\mathbb{R}[x]$. Show that

$$\deg(f(x)g(x)) = \deg(f(x)) + \deg(g(x)).$$

(b) (**5 marks**) Let $a(x)$ and $b(x)$ be polynomials of $\mathbb{R}[x]$ such that $a(x) \mid b(x)$ and $b(x) \mid a(x)$. Show that

$$a(x) = cb(x)$$

for some non-zero $c \in \mathbb{R}$.

(c) (**5 marks**) Find an integer $n \in \mathbb{N}$, and two polynomials $c(x), d(x) \in \mathbb{Z}_n[x]$ such that

$$\deg(c(x)d(x)) \neq \deg(c(x)) + \deg(d(x)).$$

3. (**12 marks**) Let $f(x)$ and $g(x)$ be polynomials in $\mathbb{Z}_5[x]$ defined by

$$f(x) = x^4 + 2x^3 + 4x^2 + 2x + 3, \quad g(x) = 4x^3 + 2x^2 + 4x + 2,$$

where for simplicity, we denote $\bar{a}$ by a for $\bar{a} \in \mathbb{Z}_5$.

(a) (**6 marks**) Find quotient $q(x)$ and remainder $r(x)$ when $f(x)$ is divided by $g(x)$.

(b) (**6 marks**) Find a greatest common divisor of $f(x)$ and $g(x)$ and find polynomials $u(x)$ and $v(x)$ such that

$$d(x) = f(x)u(x) + g(x)v(x).$$

4. (**8 marks**) Consider the following set K of matrices:

$$K = \left\{ \begin{bmatrix} x & x \\ 0 & 0 \end{bmatrix} : x \in \mathbb{R}, x \neq 0 \right\}.$$

Prove that K is a group under matrix multiplication.