DEPARTMENT OF MATHEMATICS

MATHS 255 | Assignment 6 | Due: 16 September, 2003

Note: Please deposit your answers in the appropriate box outside the Student Resource Centre in the basement of the Mathematics/Physics building **by 4 pm on the due date.** Late assignments will not be marked. Use a Mathematics Department cover sheet which is available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. Determine whether or not the following subsets of the poset ($\mathbf{R}, \leq$) are order isomorphic.

   ["Determine" means prove your answer]

   (a). $\mathbf{R}$, $\mathbf{R}^+$ (i.e. the positive real numbers).

   (b) $\mathbf{R}$, (0,1) (i.e the open interval).

   (c) $\left\{\frac{1}{n} : n \in \mathbf{Z}\setminus\{0\}\right\}$, $\mathbf{Z}\setminus\{0\}$.

2. Let $S = \mathbf{Q}\setminus\{1\}$.

   (a) Show that * defined as follows is a binary operation on S: $a*b = a+b-ab$. [You must show that if $a,b \in S$ then $a*b \in S$.]

   (b) Show that * is commutative and associative.

   (c) Find an identity element e under the operation *.

   (d) Prove that your answer to (c) is unique (i.e. that if e, f are both identity elements under the operation *, then $e = f$.

   (e) Show that every element of S has an "inverse", i.e. $\forall x \in S\ \exists y \in S,\ x*y = e$ (where e is the unique identity element from (c) and (d) above).

3. Prove that the product of any four consecutive positive integers is divisible by 12.

4. Use the Euclidean algorithm to find $d = \gcd(m,n)$ and to find integers u,v such that $d = mu + nv$, and use prime factorization to find $\mathrm{lcm}(m,n)$ if $m = 4635$ and $n = 17061$.