

Note: Please deposit your answers in the appropriate box outside the Student Resource Centre in the basement of the Mathematics/Physics building **by 4 pm on the due date**. Late assignments will not be marked. Use a Mathematics Department cover sheet which is available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. Determine (with proofs or counterexamples) whether or not the following relation f defined on $\mathcal{Q} \times \mathcal{Q}$ by $f = \{(m/n, n/m) : n, m \text{ non-zero integers}\} \times \{(0,1)\}$ is

(a). a function from the set \mathcal{Q} to the set \mathcal{Q} . (\mathcal{Q} =rational numbers.)

(b). one-to-one

(c). onto

2. Let A and B be sets, and let $S \subseteq A \times B$. We define the *projection functions* on S by

$$\pi_1 : S \rightarrow A, \pi_2 : S \rightarrow B \text{ by } \pi_1(a,b) = a, \pi_2(a,b) = b.$$

(a) Show (using examples) that both of these functions are not necessarily either one-to-one or onto (i.e. four things to show).

(b) Assume further that S is a function from A to B . Must it be true that π_1 is one-to-one? π_1 is onto? π_2 is one-to-one? π_2 is onto? Prove your answers.

3. Let $f : A \rightarrow B$ be a function, X and Y subsets of A , U and V subsets of B .

(a) Prove that $f^{-1}(U) \setminus f^{-1}(V) = f^{-1}(U \setminus V)$.

(b) Prove that $f(X) \setminus f(Y) \subseteq f(X \setminus Y)$.

(c) Show that $f(X) \setminus f(Y) = f(X \setminus Y)$ for all X and Y subsets of A if and only if f is one-to-one.