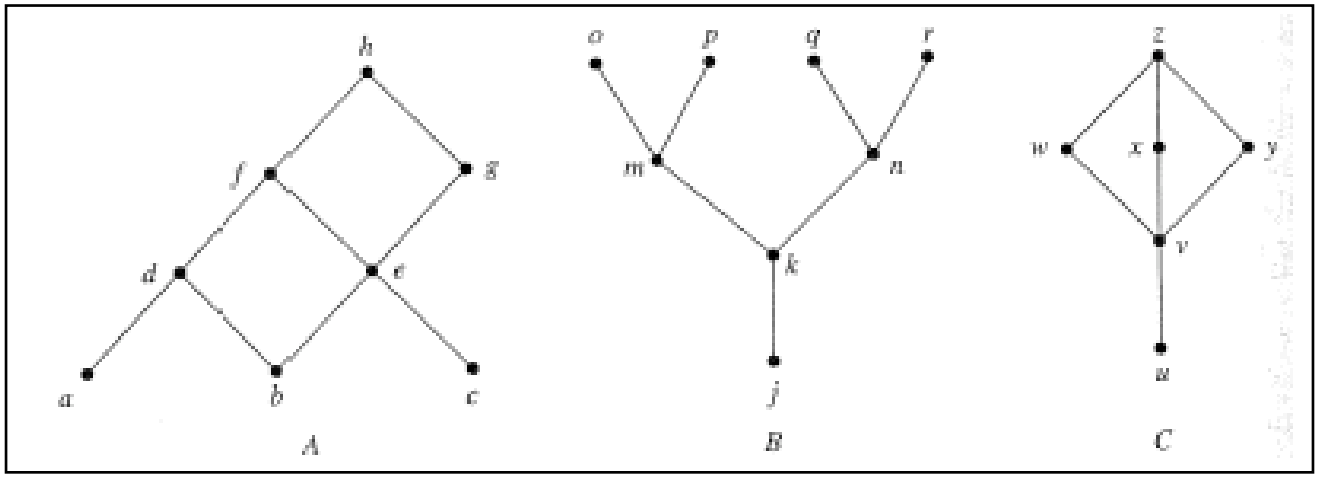
DEPARTMENT OF MATHEMATICS

MATHS 255 Assignment 4 Due: 19 August, 2003

Note: Please deposit your answers in the appropriate box outside the Student Resource Centre in the basement of the Mathematics/Physics building **by 4 pm on the due date.** Late assignments will not be marked. Use a Mathematics Department cover sheet which is available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. A relation $\sqcup$ on a set S is *antireflexive* if for each x in S, $x \sqcup x$ is false. A relation which is antireflexive and transitive is called a *quasi-order relation*. Prove that if $\leq$ is a partial ordering on S, then the relation $<$ defined by $x<y$ if and only if $x \leq y$ and $x \neq y$ is a quasi-order relation.

2. The figures show the lattice diagrams of three posets.



(a) What are the maximal elements of these posets?
(b) What are the minimal elements of these posets?
(c) Find all smallest elements of these posets.
(d) Which elements cover the element e?
(e) Find all upper bounds and all lower bounds of the set $\{p,n\}$.
(f) Find each of the following if it exists:
lub$\{d,c\}$. lub$\{w,y.v\}$. lub$\{p,k\}$. glb$\{a,g\}$. glb$\{p,n\}$
(g) Write down *all* of the pairs in the partial ordering represented by figure C.

3. Suppose that X is a poset with partial ordering $\leq$, and suppose that A is a subset of X. Let U be the set of all upper bounds of A. Show that if U has a greatest lower bound g, then g is a least upper bound of A.

4. (a) Let $\sim$ be the relation defined on the complex numbers $\boldsymbol{C}$ by $z \sim w$ if $|z - i| = |w - i|$. Then $\sim$ is an equivalence relation. Describe geometrically the equivalence class containing $z = 4 + 4i$.

(b) Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 1 & 2 \end{bmatrix}$ and let ~ be the relation defined on the plane $\mathbf{R}^2$ by $(x,y) \sim (z,w)$ if $A\begin{bmatrix} x \\ y \end{bmatrix} = A\begin{bmatrix} w \\ z \end{bmatrix}$. Then ~ is an equivalence relation. Describe geometrically the equivalence class containing the point (a,b). How does it relate to Nul$A$ (the nullspace of A)?