NB: Please deposit your solutions in the appropriate box **by 4pm on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a blue Mathematics department cover sheet.

1. [*8 marks*]

Find a cubic expression for the sum $\sum_{i=1}^{n} 3(4i^2 + 2i - 1)$, and prove it using induction.

2. [*7 marks*]

Use induction to prove the following statement is true when the set A is sufficiently large.

$$|\mathcal{P}(A \times A)| > |\mathcal{P}(A) \times \mathcal{P}(A)|$$

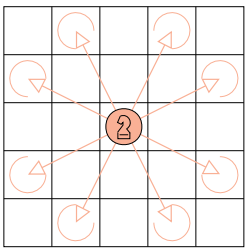
3. [*7 marks*]

Use induction to prove that $4|(7^{n+2} - 3^n)$ for any natural number n.

4. [*10 marks*]

In the game of chess a knight moves on the board in one of eight directions as shown in the figure below.

Use induction to show there is a sequence of knight moves from any square to any other square on any $n \times n$ chessboard with $n \geq 4$.



5. [*8 marks*]

Let $\lhd$ be the relation on $\mathbb{R}$ defined by $x \lhd y$ if and only if $|x - y| < 1$.

(a) Is $\lhd$ reflexive?

(b) Is $\lhd$ symmetric?

(c) Is $\lhd$ antisymmetric?

(d) Is $\lhd$ transitive?

For each part, give either a proof or a counterexample.

TOTAL MARKS: 40