NB: Please deposit your solutions in the appropriate box **by 4pm on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a blue Mathematics department cover sheet.

1. [*8 marks*] Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function for which $f(x) \neq 0$ for any $x \in \mathbb{R}$. Show that, for any $k \in \mathbb{R}$, there exists a unique function $g_k : \mathbb{R} \to \mathbb{R}$ so that $f(x) \cdot g_k(x) = k$ for every $x \in \mathbb{R}$.

[Hint: The proof requires two parts. First show that the function $g_k(x)$ exists, and then show that it is unique.]

2. [*4 marks*] Explain why the the above statement is not true if $f(\alpha) = 0$ for some root α of f.

[Hint: Consider what might 'go wrong' with different values of k.]

3. [*13 marks*] A function $f : \mathbb{R} \to \mathbb{R}$ is called *odd* if for all x, $f(-x) = -f(x)$. Similarly, a function is called *even* if for all x, $f(-x) = f(x)$.

Consider the following statements:

(a) If $f(x)$ and $g(x)$ are even functions, then $f(x) \cdot g(x)$ is an even function.

(b) If $f(x)$ and $g(x)$ are odd functions, then $f(x) + g(x)$ is an odd function.

(c) There is a unique function which is both even and odd.

For each statement, give a proof (in any form you choose). State the type of proof you use.

Also, for (a) and (b), either prove that the converse is true, or give a counterexample to the converse.

4. [*10 marks*] Let A be the set $\{1, 2, 3, 4\}$ and B be the set $\{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\}$.

(a) For each of the following statements about the sets A and B, state whether it is true or false.

(i)	$2 \in A$	(iv)	$B \subseteq \mathcal{P}(A)$	(vii)	$A \cap B = \emptyset$
(ii)	$A \subseteq B$	(v)	$\{1, 3\} \subset B$		
(iii)	$A \in B$	(vi)	$C \in B \implies C \subseteq A$	(viii)	$\bigcup_{C \in B} C = A$

(b) Use set builder notation to describe the B as a subset of $\mathcal{P}(A)$.

5. [*6 marks*] Given two sets A and B we define $A \oplus B = \{x : x \in A \text{ or } x \in B, \text{ but not both}\}$.

Show that for any sets, the operation $\oplus$ is commutative and associative.

TOTAL MARKS: 40