NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

1. (**5 marks**) Determine with reasons that whether or not there is a subgroup of order 5 in D_4.

2. (**13 marks**) Let F be a field. For $x, y \in F$ with $y \neq 0$, denote xy^{-1} by x/y. Prove that for all $a, b, c, d \in F$ the following hold.

(a) (**3 marks**) If $ab = 0$ then either $a = 0$ or $b = 0$.

(b) (**3 marks**) $(-a)b = -(ab)$ and $(-a)(-b) = ab$.

(c) (**3 marks**) If $b, d \neq 0_F$, then $a/b + c/d = (ad + bc)/bd$.

(d) (**2 marks**) If $b, d \neq 0_F$, then $(a/b)(c/d) = ac/bd$

(e) (**2 marks**) If $b, c, d \neq 0_F$, then $(a/b)/(c/d) = ad/bc$

3. (**14 marks**)

(a) Let F be an ordered field. Prove that for all $a, b, c \in F$ the following hold.

(i) (**3 marks**) If $a < b$ and $c < 0$ then $bc < ac$.

(ii) (**3 marks**) If $a < b$, then $-b < -a$. In particular, $a < 0$, then $0 < -a$.

(iii) (**4 marks**) If $ab > 0$ (that is, $0 < ab$), then either both $a, b > 0$ or both $a, b < 0$.

(b) (**4 marks**) Show that $\mathbb{C}$ is not an ordered field.

4. (**8 marks**) Let S and T be two non-empty sets of real numbers such that $x \leq y$ for all $x \in S$ and $y \in T$. Let lub S be the least upper bound of S and glb T be the greatest lower bound of T. Show that both lub S and glb T exist and

$$\text{lub } S \leq \text{glb } T.$$