NB: Please deposit your solutions in the appropriate box **by 4pm on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a blue Mathematics department cover sheet.

1. [*4 marks*] Classify which of the following sentences are statements, which are predicates, and which are neither.

(a) If n is even, then $n^2 + 1$ is odd.

(b) $11,111,111$ is a prime number.

(c) Every number that is a multiple of 12 is a multiple of 4.

(d) Solve this quadratic equation.

2. [*6 marks*] Translate the statements and predicates of Question 1 into symbols. Clearly define what each of your predicates and variables means.

3. [*2 marks*] Read Chapter 0 of your textbook, "Chapter Zero" by Carol Shumacher. It contains a discussion about the importance of precision in mathematical language. Explain the difference between the two statements about poisons on page 5: "For every poison there is a chemical that is the antidote" and "There is a chemical that is the antidote for every poison".

4. [*8 marks*] Construct truth tables for each of the following statements.

(a) $(A \wedge B) \implies (A \vee B)$

(b) $(A \implies B) \implies \neg A$

(c) $(A \implies B) \implies (\neg B \implies \neg A)$

(d) $A \wedge ((A \implies B) \wedge (A \implies \neg B))$

5. [*2 marks*] Which of the statements in Question 4 are tautologies?

6. [*2 marks*] Translate a contradiction listed in Question 4 into an English sentence. Explain why it is **intuitively** a contradiction. Use any definition for the variables you like, but remember that a contradiction is always false, regardless of the definitions given to its variables.

7. [*16 marks*] For any positive integers m and n, let $A(m, n)$ be the statement

"If n is a factor of m^2, then n is a factor of m."

(a) Write down the contrapositive of $A(m, n)$. Call this $B(m, n)$.

(b) Write down the negation of $A(m, n)$. Call this $C(m, n)$.

(c) Write down the converse of $A(m, n)$. Call this $D(m, n)$.

(d) Which of $A(m, n)$, $B(m, n)$ and $D(m, n)$ are true for all n, with some specified value for m? (That is, $(\exists m)(\forall n) \ldots$).

(e) Which of $A(m, n)$, $C(m, n)$ and $D(m, n)$ are true for all m and n?

For each answer in (d) and (e), give a proof or a counterexample.

Marks will be allocated in this question for clear, precise mathematical English.

TOTAL MARKS: 40