1. (a) "17 is a prime number" is a statement, which we would translate as $P(17)$ (where $P(n)$ denotes "n is prime").

(b) "If n is a prime number then n is odd" is a predicate, which we would translate as $P(n) \implies O(n)$ (where $O(n)$ denotes "n is odd").

(c) "Is 13 a prime number?" is neither a statement nor a predicate.

(d) "Every even number is the sum of two odd numbers" is a statement, which we would symbolise as $(\forall n)(E(n) \implies (\exists m,k)(O(m) \wedge O(k) \wedge S(m,k,n)))$ (where $E(n)$ denotes "n is even" and $S(m,k,n)$ denotes "$m+k=n$").

2. (a) We have the truth table

A	B	$(A$	$\implies$	$\sim$	$B)$	$\wedge$	$(A$	$\implies$	$B)$
T	T	T	F	F	T	F	T	T	T
T	F	T	T	T	F	F	T	F	F
F	T	F	T	F	T	T	F	T	T
F	F	F	T	T	F	T	F	T	F

From this we see that the main column contains a mixture of Ts and Fs, so the statement is neither a tautology nor a contradiction.

(b) We have the truth table

A	B	$(A$	$\implies$	$B)$	$\wedge$	$(\sim$	A	$\implies$	$B)$
T	T	T	T	T	T	F	T	T	T
T	F	T	F	F	F	F	T	T	F
F	T	F	T	T	T	T	F	T	T
F	F	F	T	F	F	T	F	F	F

Again we see that this is neither a tautology nor a contradiction.

(c) We have the truth table

A	B	$(A$	$\iff$	$B)$	$\vee$	$(A$	$\iff$	$\sim$	$B)$
T	T	T	T	T	T	T	F	F	T
T	F	T	F	F	T	T	T	T	F
F	T	F	F	T	T	F	T	F	T
F	F	F	T	F	T	F	F	T	F

This time the main column has all Ts so this is a tautology.

(d) We have the truth table

A	B	$\sim$	$(B$	$\implies$	$A)$	$\implies$	$\sim$	A
T	T	F	T	T	T	T	F	T
T	F	F	F	T	T	T	F	T
F	T	T	T	F	F	T	T	F
F	F	F	F	T	F	T	T	F

Again we see that this statement is a tautology.

3. (a) We have the truth table

A	B	$(A$	$\implies$	$B)$	$\iff$	$\sim$	$(A$	$\wedge$	$\sim$	$B)$
T	T	T	T	T	T	T	T	F	F	T
T	F	T	F	F	T	F	T	T	T	F
F	T	F	T	T	T	T	F	F	F	T
F	F	F	T	F	T	T	F	F	T	F

We see that the main column contains only Ts so the statement is a tautology.

(b) We have the truth table

A	B	$(\sim$	A	$\implies$	$(B$	$\wedge$	$\sim$	$B))$	$\iff$	A
T	T	F	T	T	T	F	F	T	T	T
T	F	F	T	T	F	F	T	F	T	T
F	T	T	F	F	T	F	F	T	T	F
F	F	T	F	F	F	F	T	F	T	F

Again we see that the main column contains only Ts so the statement is a tautology.

4. (a) The contrapositive of $A(n)$ is "If $n^2 - 1$ is odd then n is even".

(b) The converse of $A(n)$ is "If $n^2 - 1$ is even then n is odd".

(c) The negation of $A(n)$ is "n is odd and $n^2 - 1$ is odd".

(d) $A(n)$ is true for some $n \in \mathbb{N}$: for example, $A(1)$ is true (since 1 is odd and $0 = 1^2 - 1$ is even).

(e) $A(n)$ is true for all $n \in \mathbb{N}$. We give a direct proof. Suppose n is odd. Then $n = 2k+1$ for some $k \in \mathbb{Z}$, and so $n^2 - 1 = 4k^2 + 4k + 1 - 1 = 2(2k^2 + 2k)$, and $2k^2 + 2k \in \mathbb{Z}$, so $n^2 - 1$ is even.

(f) By (d) and (e), the contrapositive is true for some $n \in \mathbb{N}$, and indeed for all $n \in \mathbb{N}$, since it is equivalent to $A(n)$ itself.

(g) The converse of $A(n)$ is true for some $n \in \mathbb{N}$: for example the converse of $A(2)$ is (vacuously) true since $2^2 - 1 = 3$ is not even.

(h) The converse of $A(n)$ is true for all $n \in \mathbb{N}$. We give a proof by contraposition. So suppose that n is not odd. Then n is even, so $n = 2k$ for some $k \in \mathbb{Z}$, and so $n^2 - 1 = 4k^2 - 1 = 2(2k^2) - 1$, and $2k^2 \in \mathbb{Z}$, so $n^2 - 1$ is odd. Hence, by contraposition, if $n^2 - 1$ is even then n is odd.