1. (a) Let $x, y \in A$ with $x\rho y$ and $y\rho x$. Then, by tranisitivity, $x\rho x$. But this contradicts irreflexivity. So there are no such x and y and hence, vacuously, ρ is antisymmetric.

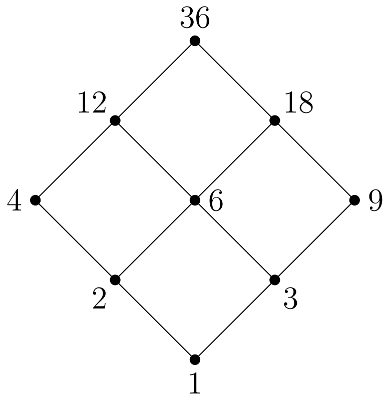
(b) We must show that $\preceq$ is reflexive, antisymmetric and transitive.

Reflexive: Let $x \in A$. Then $x = x$ so $x \preceq x$.

Antisymmetric: Let $x, y \in A$ with $x \preceq y$ and $y \preceq x$. Suppose, for a contradiction, that $x \neq y$. Then we must have $x \preceq y$ and $y \preceq x$. But then, by (a), we have $x = y$ after all, a contradiction. So we must have $x = y$, as required.

Transitive: Let $x, y, z \in A$ with $x \preceq y$ and $y \preceq z$. Then $x \ \rho \ y$ or $x = y$, and $y \ \rho \ z$ or $y = z$. If $x = y$ then since $y \preceq z$ we have $x \preceq z$. Similarly if $y = z$ then since $x \preceq y$, $x \preceq z$. So suppose $x \neq y \neq z$. Then $x \ \rho \ y$ and $y \ \rho \ z$, so $x \ \rho \ z$ (since ρ is transitive), so $x \preceq z$ as required.

2. We have the diagram



3. We must show that r is an upper bound for $A \cup B$, and that it is the least upper bound.

upper bound: let $x \in A \cup B$. Then $x \in A$ or $x \in B$. If $x \in A$ then $x \preceq p$ (since p is an upper bound for A) and $p \preceq r$ (since r is an upper bound for $\{p, q\}$) so $x \preceq r$. Similarly if $x \in B$ then $x \preceq q \preceq r$, so $x \preceq r$. So either way we must have $x \preceq r$, so r is an upper bound for $A \cup B$.

least: Let s be an upper bound for $A \cup B$. For every $x \in A$, we have $x \in A \cup B$, so $x \preceq s$. Thus s is an upper bound for A, and $p = \sup A$, so $p \preceq s$. Similarly, if $y \in B$ then $y \in A \cup B$ so $y \preceq s$. Thus s is an upper bound for B, so $q \preceq s$. Thus s is an upper bound for $\{p, q\}$, so $\sup\{p, q\} \preceq s$, i.e. $r \preceq s$, as required.