1. Let $a, b \in \mathbb{R}$. Prove that $|a + b| \leq |a| + |b|$. [Hint: add together the two inequalities $-|a| \leq a \leq |a|$ and $-|b| \leq b \leq |b|$.]

2. Use the result of Question 1 to deduce the *triangle inequality*: for any $x, y, z \in \mathbb{R}$ we have $|x - z| \leq |x - y| + |y - z|$. Recall that $|x - y|$ can be thought of as the distance from x to y: why do you think this result is called the triangle inequality?

3. Recall that a sequence (s_n) *converges to* L if for all $\varepsilon > 0$ there is some $N \in \mathbb{N}$ such that for all $n \in \mathbb{N}$ with $n \geq N$, $|s_n - L| < \varepsilon$. What does it mean to say that (s_n) does not converge to L?

4. Show that if a sequence (s_n) in $\mathbb{R}$ has a limit, then the limit is unique.

5. Let (a_n) and (b_n) be sequences in $\mathbb{R}$. Suppose that $a_n \to L$ and $b_n \to M$ as $n \to \infty$. Define a new sequence (c_n) by declaring that $c_n = a_n + b_n$. Show that $c_n \to L + M$ as $n \to \infty$.