

1. Let  $a(x) = x^3 - x^2 - 15x - 25$ .
  - (a) Evaluate  $a(5)$ .
  - (b) Use the factor theorem to express  $a(x)$  as a product of irreducible polynomials. [Hint: the factors will turn out to be one linear and one quadratic, and you should explain why the quadratic factor is irreducible.]
  
2. Let  $(G, *)$  be a group.
  - (a) Show that for any  $x, y \in G$ ,  $(x * y)^{-1} = y^{-1} * x^{-1}$ .
  - (b) Show that for any  $x, y \in G$ ,  $x * y = y * x$  iff  $(x * y)^{-1} = x^{-1} * y^{-1}$ .
  
3. Let  $(G, *)$  be a group, and fix  $g \in G$ . Define a function  $f_g : G \rightarrow G$  by  $f_g(x) = g^{-1} * x * g$ . Show that  $f_g$  is one-to-one and onto, and that for any  $x, y \in G$  we have  $f_g(x) * f_g(y) = f_g(x * y)$ . [To make things easier to write, from now on we may omit the  $*$  and write this as  $f_g(x)f_g(y) = f_g(xy)$ , and so on. We will also often omit brackets to avoid writing out the rearrangements needed using associativity.]