Corrections to Assignment 6: please note two corrections to Assignment 6. In question 1(b), replace "Suppose $*$ has an identity element…" with "Suppose $*$ is associative and has an identity element…". In question 3(e), replace "$gl = axbn + byan$" with "$gl = axbn + byam$".

1. To show that for integers x and y we have $y = -x$, we have to show that $x + y = 0$. Use this to prove from the axioms that

(a) for any $x \in \mathbb{Z}$, $-(-x) = x$; and

(b) for any $x \in \mathbb{Z}$, $-x = (-1) \cdot x$. [You may assume the result proved in lectures that $0 \cdot x = 0$ for all $x \in \mathbb{Z}$.]

2. Show that if $a, b, c, x, y \in \mathbb{Z}$ with $c \mid a$ and $c \mid b$ then $c \mid ax + by$.

3. Recall that if $r, s \in \mathbb{Z}$, we say that r and s are *relatively prime* if the only positive common divisor of r and s is 1, in other words if $\gcd(r, s) = 1$.

(a) Show that if r and s are relatively prime, then so are r and $r + s$.

(b) Show that if there exist $x, y \in \mathbb{Z}$ with $rx + sy = 1$ then r and s are relatively prime.

4. Recall that by definition, $x + 1$ is an immediate successor of x, in other words $x < x + 1$ and there is no w with $x < w < x + 1$. Use this to show that if $x, y \in \mathbb{Z}$ with $x + 1 = y + 1$ then $x = y$. [Hint: use a proof by contradiction: if $x \neq y$ then WLOG $x < y$.]