1. A relation ρ on a set A is said to be *irreflexive* if for all $x \in A$, $x \not\rho\, x$. It is an *SPO* if it is irreflexive and transitive.

   (a) Show that if ρ is an SPO then ρ is antisymmetric.

   (b) Show that if ρ is an SPO then the relation $\preceq$ defined by declaring that

   $$x \preceq y \iff (x \rho y \vee x = y)$$

   is a partial order.

2. Let $A = \{\, n \in \mathbb{N} : n \mid 36 \,\}$. Draw a lattice diagram for the poset $(A, |)$.

3. Let $(S, \preceq)$ be a poset, and let $A, B \subseteq S$, $p, q, r \in S$. Suppose that

   - $p = \sup A$;
   - $q = \sup B$; and
   - $r = \sup\{p, q\}$.

   Show that r is a least upper bound for $A \cup B$. [Hint: in fact, for $s \in S$, s is an upper bound for $A \cup B$ iff s is an upper bound for $\{p, q\}$. You should **not** use this fact without proof, but it may guide you in finding your proof that $r = \sup(A \cup B)$.]