1. A function $f : \mathbb{R} \to \mathbb{R}$ is *strictly monotone* if $(\forall x, y \in \mathbb{R})(x < y \implies f(x) < f(y)) \vee (\forall x, y \in \mathbb{R})(x < y \implies f(x) > f(y))$. It is *one-to-one* if $(\forall x, y \in \mathbb{R})(f(x) = f(y) \implies x = y)$.

What is wrong with the following "proof" that if f is one-to-one then f is strictly monotone?

> Suppose f is one-to-one. Let $x, y \in \mathbb{R}$ with $x < y$. Then $x \neq y$, and by contraposition $x \neq y \implies f(x) \neq f(y)$, so $f(x) \neq f(y)$. So $f(x) < f(y)$ or $f(y) < f(x)$. So either $x < y \implies f(x) < f(y)$ or $x < y \implies f(x) < f(y)$. Hence f is strictly monotone.

[Not only is the proof wrong, but the result is not true. For example, the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 1/x$ for $x \neq 0$ and $f(0) = 0$ is one-to-one but not strictly monotone.]

2. Which of the following are true and which are false?

(a) $\emptyset \in \emptyset$.

(b) $\emptyset \subseteq \emptyset$.

(c) $\emptyset \in \{\emptyset\}$.

(d) $\emptyset \subseteq \{\emptyset\}$.

(e) $\emptyset = \{\emptyset\}$.

(f) $\{\emptyset\} \in \emptyset$.

(g) $\{\emptyset\} \in \{\emptyset\}$.

(h) $\{\emptyset\} \subseteq \emptyset$.

(i) $\{\emptyset\} \subseteq \{\emptyset\}$.

3. For $n \in \mathbb{N}$, let $A_n = \{1, 2, \ldots, n\}$.

(a) Find $\bigcup_{n \in N} A_n$.

(b) Find $\bigcap_{n \in N} A_n$.

4. Let S be a set.

(a) Show that $\bigcup_{a \in S} \{a\} = S$.

(b) Show that $\bigcup_{A \in \mathcal{P}(S)} A = S$.

5. As before, for $n \in \mathbb{N}$ let $A_n = \{1, 2, \ldots, n\}$. Show that $\mathcal{P}(A_{n+1}) = \mathcal{P}(A_n) \cup \{\, S \cup \{n+1\} : S \in \mathcal{P}(A_n) \,\}$.