1. We use the extended Euclid's Algorithm:

n	x	y	
35	1	0	r_1
25	0	1	r_2
10	1	-1	$r_3 = r_1 - r_2$
5	-2	3	$r_4 = r_2 - 2r_3$
0	5	-7	$r_5 = r_3 - 2r_4$

From this we see that $\gcd(35, 25) = 5 = 35 \cdot (-2) + 25 \cdot 3$. Since $5 \nmid 12$, $35x + 25y = 12$ has no solutions. Since $15 = 5 \cdot 3$, we have $15 = 35 \cdot (-6) + 25 \cdot 9$, so the general solution in $15 = 35(-6 - t\frac{25}{5}) + 25(9 + t\frac{35}{5}) = 25(-6 - 5t) + 25(9 + 7t)$ for $t \in \mathbb{Z}$, in other words $x = -6 - 5t$, $y = 9 + 7t$ for $t \in \mathbb{Z}$.

2. (a) Again, we use the extended Euclid's Algorithm:

n	x	y	
15	0	1	r_1
6	1	0	r_2
3	-2	1	$r_3 = r_1 - r_2$
0	5	-2	$r_4 = r_2 - 2r_3$

From this we see that $\gcd(6, 15) = 3 = 6 \cdot (-2) + 15 \cdot 1$, and $9 = 3 \cdot 3$, so the particular solution of the equation is $9 = 6 \cdot (-6) + 15 \cdot 3$ and the general solution is $x = -6 - \frac{15}{3}t = -6 - 5t$, $y = 3 + \frac{6}{3}t = 3 + 2t$ for $t \in \mathbb{Z}$.

(b) From (a) we know that $\overline{6} \cdot_{15} \overline{x} = \overline{9}$ iff $\overline{x} = \overline{-6 - 5t} = \overline{-6} - \overline{5t}$ for some $t \in \mathbb{Z}$. But there are only three values for $\overline{5t}$, namely $\overline{0}$, $\overline{5}$ and $\overline{10}$. So the solutions are $\overline{-6}$, $\overline{-6} - \overline{5}$ and $\overline{-6} - \overline{10}$, in other words $\overline{-6}$, $\overline{-11}$ and $\overline{-16}$, or $\overline{9}$, $\overline{4}$ and $\overline{14}$.

3. Suppose a and c are relatively prime and b and c are relatively prime. Then there exist $x, y, x', y' \in \mathbb{Z}$ with

$$1 = ax + cy \tag{1}$$

$$1 = bx' + cy' \tag{2}$$

Mutliplying (2) by a gives $a = abx' + acy'$. Substituting this value of a in to (1) gives $1 = (abx' + acy')x + cy = (ab)(xx') + c(ay'x + y)$. Since $xx' \in \mathbb{Z}$ and $ay'x + y \in \mathbb{Z}$, this shows us that ab and c are relatively prime.