

1. (a) We can write this as “For every natural number  $n$ , if  $n$  is odd then  $n$  is the sum of two prime numbers”, or even as “For every natural number  $n$ , if  $n$  is odd then there exist prime numbers  $m$  and  $k$  such that  $n = m + k$ .”
- (b) We can translate it into symbols as  $(\forall n \in \mathbb{N})(O(n) \implies (\exists m)(\exists k)(P(m) \wedge P(k) \wedge S(m, k, n))$ .
2. (a) “If  $2n$  is even then  $n$  is even.”
- (b) “If  $2n$  is odd then  $n$  is odd.”
- (c) “ $n$  is even and  $2n$  is odd.” [Or, “ $n$  is even but  $2n$  is odd.”]

3. Let  $x$  be an integer.

[We need to give two proofs: that if  $x$  is even then  $x^3$  is even, and conversely that if  $x^3$  is even then  $x$  is even. We use a direct proof for the first and proof by contraposition for the second.]

Suppose first that  $x$  is even. Then  $x = 2y$  for some  $y \in \mathbb{Z}$ . So  $x^3 = (2y)^3 = 8y^3 = 2(4y^3)$ , and  $4y^3 \in \mathbb{Z}$ , so  $x^3$  is even.

Conversely, suppose that  $x$  is not even. Then  $x$  is odd, so  $x = 2u + 1$  for some  $u \in \mathbb{Z}$ . So

$$\begin{aligned}x^3 &= (2u + 1)^3 \\&= (2u)^3 + 3(2u)^2 + 3(2u) + 1 \\&= 8u^3 + 12u^2 + 6u + 1 \\&= 2(4u^3 + 6u^2 + 3u) + 1,\end{aligned}$$

so  $x^3$  is odd. Hence, by contraposition, if  $x^3$  is even then  $x$  is even.