1. Find all solutions of the Diophantine equations

$$35x + 25y = 12 \qquad \text{and} \qquad 35x + 25y = 15.$$

2. (a) Find all solutions of the Diophantine equation $6x + 15y = 9$.

   (b) Use your answer to (a) to find all solutions of the equation $\bar{6} \cdot_{15} \bar{x} = \bar{9}$ in $\mathbb{Z}_{15}$.

3. Recall that for $a, b \in \mathbb{Z}$, a and b are relatively prime iff there exist $x, y \in \mathbb{Z}$ with $ax + by = 1$. Use this fact to prove that if a and c are relatively prime, and b and c are relatively prime, then ab and c are relatively prime. [Hint: write equations expressing 1 as a linear combination of a and c, and of b and c. Mutliply the second equation by a and substitute this value of a into the first equation. Now rearrange to the form $1 = (ab)x^* + cy^*$.]