1. Let $f : A \to B$ be a function. Define a new function $F : \mathcal{P}(A) \to \mathcal{P}(B)$ by declaring that, for $S \subseteq A$,

$$F(S) = \{ f(a) : a \in S \}.$$

Show that F is one-to-one if and only if f is one-to-one.

2. Let $f : A \to B$ be a bijection. This tells us that $f^{-1} : B \to A$ exists and is a function. Prove that f^{-1} is also a bijection.

3. Let $(A, \preceq_A)$ and $(B, \preceq_B)$ be posets and $f : A \to B$ an order-isomorphism. Let $x \in A$. Show that x is maximal in A iff $f(x)$ is maximal in B. [Recall that x is *maximal* if there does not exist any $z \in A$ with $x \prec_A z$: this is equivalent to saying that if $x \preceq_A z$ then $x = z$.]

4. Let $A = \{ 1 - \frac{1}{n} : n \in \mathbb{N} \}$ and let $B = A \cup \{1\}$, both ordered with the usual $\leq$ order they get as subsets of $\mathbb{R}$. Use the result of the previous question to show that A and B are **not** order-isomorphic.