NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

1. The symmetry group for a rectangle which is not a square has four elements: the identity R_0, rotation R_{180} through 180°, and flips V and H about vertical and horizontal axes respectively. Compute the Cayley Table for this group.

2. In the full symmetric group S_6, let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 6 & 5 & 4 & 1 \end{bmatrix}$.

(a) Write α as a product of disjoint cycles.

(b) Let β be the cycle (1 2 3), so that $\beta^{-1} = (3\ 2\ 1)$. Calculate $\beta\alpha\beta^{-1}$.

3. Let G, H and K be groups and let $f : G \to H$ and $g : H \to K$ be homomorphisms. Show that $g \circ f$ is a homomorphism from G to K.

4. Let G and H be groups and let $f : G \to H$ be a homomorphism. Show that f is one-to-one if and only if $f^{-1}(\{e_H\}) = \{e_G\}$.

5. Let G be a finite group with identity e.

(a) Let $x \in G \setminus \{e\}$. Show that $\{e, x\}$ is a subgroup of G if and only if $x^2 = e$.

(b) Show that G has an element x with $x \neq e$ and $x^2 = e$ if and only if $|G|$ is even. [Hint: in one direction, we can use Lagrange's Theorem and part (a). In the other direction, suppose $|G|$ is even. Put $A = \{ g \in G : g^2 \neq e \}$. Show that if $g \in A$ then $g \neq g^{-1}$ and $g^{-1} \in A$. Explain why this means that $|A|$ must be even. Then $|G \setminus A|$ is also even, and $e \in G \setminus A$, so there is some $x \in G \setminus A$ with $x \neq e$.]