NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

1. Find all solutions to the following Diophantine equations:

(a) $35x + 12y = 16$.

(b) $30x + 12y = 15$.

(c) $30x + 12y = 18$.

2. For $a \in \mathbb{Z}$, $n \in \mathbb{N}$, let $r_n(a)$ denote the remainder when a is divided by n, in other words the integer with $0 \leq r_n(a) < n$ such that for some $q \in \mathbb{Z}$, $a = qn + r_n(a)$.

Prove that for $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$, $a \equiv b \pmod{n}$ if and only if $r_n(a) = r_n(b)$.

3. Solve the equation $\overline{33} = \overline{47} \cdot_{250} \overline{x}$ in $\mathbb{Z}_{250}$.

4. Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. Show that if the equation $\overline{a} = \overline{b} \cdot_n \overline{x}$ has a unique solution in $\mathbb{Z}_n$ then b and n are relatively prime. [Hint: prove the contrapositive, in other words show that if b and n are not relatively prime then the equation has either no solutions or more than one solution. Note that the equation $a = bx + ny$ will have no solutions or infinitely many solutions: you must show that in the latter case there are solutions which are not congruent modulo n.]