NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

1. Let $f : A \to B$ be a function. Define a new function $F : \mathcal{P}(A) \to \mathcal{P}(B)$ by declaring that, for $S \subseteq A$,

$$F(S) = \{\, f(a) : a \in S \,\}.$$

Show that F is onto if and only if f is onto.

2. Let $f : A \to B$ and $g : B \to C$ be functions.

(a) Show that if $g \circ f$ is one-to-one and f is onto then g is one-to-one.

(b) Show that if $g \circ f$ is onto and g is one-to-one then f is onto.

3. Let $f : A \to B$ be a function, let Λ be a nonempty indexing set, and for each $\alpha \in \Lambda$ let S_α be a subset of B.

(a) Show that $f^{-1}(\bigcap_{\alpha\in\Lambda} S_\alpha) = \bigcap_{\alpha\in\Lambda} f^{-1}(S_\alpha)$.

(b) Show that $f^{-1}(\bigcup_{\alpha\in\Lambda} S_\alpha) = \bigcup_{\alpha\in\Lambda} f^{-1}(S_\alpha)$.

4. Suppose $(A, \preceq_A)$ and $(B, \preceq_B)$ are posets and $f : A \to B$ is a function. Show that f is strictly order preserving if and only if f is one-to one and for all $x, y \in A$,

$$x \prec_A y \iff f(x) \prec_B f(y).$$

[Recall that $x \prec_A y$ means $x \preceq_A y$ and $x \neq y$, and similarly for $\prec_B$. You may assume the result from lectures that if f is strictly order preserving then it is one-to-one.]

5. Let $f : \mathbb{N} \to \mathbb{N}$ be strictly order preserving, where $\mathbb{N}$ is give its usual ordering $\leq$. Prove that for all $n \in \mathbb{N}$, $n \leq f(n)$. [Hint: use induction and the result of the previous question.]