NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

1. Show that for all $n \in \mathbb{N}$, $3 \mid n^3 + 2n$.

2. Show that for all $n \geq 4$, $n! > 2^n$.

3. Show that if $m, n \in \mathbb{N}$ then there exist $q, r \in \mathbb{Z}$ with $0 \leq r < n$ and $m = qn + r$. [Hint: use complete induction on m. For the induction step, consider three cases—$m + 1 < n$, $m + 1 = n$ or $m + 1 > n$. In the latter case, apply the inductive hypothesis to $(m + 1) - n$.]

4. A function $f : \mathbb{Z} \to \mathbb{Z}$ is a *wibble* function if for all $n \in \mathbb{Z}$, $f(n+2) = f(n) + 2$. Suppose f and g are wibble functions, and that $f(0) = g(0)$ and $f(15) = g(15)$. Show that $f(n) = g(n)$ for all $n \in \mathbb{Z}$.

5. Let ρ be the relation on $\mathbb{Z}$ defined by $x \,\rho\, y$ if and only if $x^2 \leq y^2$.

(a) Is ρ reflexive?

(b) Is ρ symmetric?

(c) Is ρ antisymmetric?

(d) Is ρ transitive?

In each case give either a proof or a counterexample.