NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date**. Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet: these are available from outside the Resource Centre.

1. Define the function $f : \mathbb{N} \to \mathbb{Z}$ by $f(n) = n^2 + n$.

(a) Use a direct proof to show that if $m, n \in \mathbb{N}$ with $m < n$ then $f(m) < f(n)$.

(b) Use proof by contraposition to show that if $m, n \in \mathbb{N}$ with $f(m) < f(n)$ then $m < n$.

(c) A function $g : A \to B$ is *one-to-one* if, for every $a, b \in A$, if $g(a) = g(b)$ then $a = b$. Use proof by contradiction to show that f is one-to-one.

2. Let $m, c, k \in \mathbb{R}$ with $m \neq 0$. Show that the equation $mx + c = k$ has a unique solution. [Note: this requires both an existence proof and a uniqueness proof. Of course you know how to solve the equation—what I want you to do is to show that the method you have known for years really does give a solution, and that the solution it gives is the only possible one.]

3. Let S be the set $\{1, 2, 3\}$ and let A be the set $\{\, x^2 : x \in S \,\}$.

(a) Give two other descriptions of A, one using enumeration and one using set builder notation.

(b) Which of the following are true and which are false?

(i) $1 \in S$	(iv) $1 \subseteq A$	(vii) $A \in S$
(ii) $1 \subseteq S$	(v) $S \in A$	(viii) $A \subseteq S$
(iii) $1 \in A$	(vi) $S \subseteq A$	

4. Let A and B be sets. Show that the following are equivalent:

(1) $A \cap B = A$.

(2) $A \cup B = B$.

(3) $A \subseteq B$.

[Hint: we are not asserting that these are all true, but that if any one of them is true then the others are true as well. We usually prove an equivalence like this by proving (1) $\implies$ (2), (2) $\implies$ (3) and (3) $\implies$ (1), but sometimes it is easier to prove (1) $\implies$ (3), (3) $\implies$ (2) and (2 $\implies$ (1) and sometimes it is easier to prove four implications, e.g. (1) $\implies$ (2), (2) $\implies$ (1), (1) $\implies$ (3) and (3) $\implies$ (1).]

5. The *set difference* of two sets A and B is defined by

$$A \setminus B = \{\, x : x \in A \wedge x \notin B \,\}.$$

Let A be a set, Λ a non-empty indexing set, and B_α a set for each $\alpha \in \Lambda$. Show that

$$A \setminus \bigcup_{\alpha \in \Lambda} B_\alpha = \bigcap_{\alpha \in \Lambda} (A \setminus B_\alpha).$$