SURNAME:________________ FORENAMES:________________

Department of Mathematics

445.255 Principles of Mathematics

Mid-semester Test

Thursday, 9 May, 2002

Instructions

- This test contains **SIX** questions. Attempt **ALL** questions. Show **ALL** your working.
- Sign on page 2. By signing there you are stating that you are the student whose name and ID appears above your signature.
- You have 50 minutes to do the test. Total marks 50.

SURNAME:________________ FORENAMES:________________

(Capital letters please)

ID NUMBER:________________

SIGN HERE:________________

By signing here I am stating that I am the student whose name and ID appears above.

(Official use only)

QUESTION 1 (8 marks)	
QUESTION 2 (6 marks)	
QUESTION 3 (8 marks)	
QUESTION 4 (7 marks)	
QUESTION 5 (9 marks)	
QUESTION 6 (12 marks)	
Total for 6 questions (50 marks)	

1. (8 marks) Suppose a, b are some fixed integers and let $A(c)$ be the predicate:

   If $c \mid a$ and $c \mid b$ then $c^2 \mid ab$.

   (a) Write down the negation of $A(c)$.

   (b) Write down the converse of $A(c)$.

   (c) Is " for all $c \in \mathbb{N}$, $A(c)$" true or false? Prove your answer.

2. (6 marks) Let $S = \{1, 2, 3\}$. **State** (no working required) whether or not the following relation on S is reflexive, transitive, symmetric and antisymmetric. Thus state whether or not it is an equivalence relation, a partial ordering, both of these or neither. (No working required) $R_1 = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$

3. (8 marks) Let $\sim$ be a relation on $\mathbb{N}$ given by $a \sim b$ if and only if there is a prime which is a factor of both a and b. Show whether or not this is an equivalence relation.

4. (7 marks) Suppose A, B and C are sets and $f : A \to B$ and $g : B \to C$ are functions. Suppose $g \circ f : A \to C$ is one-one. Prove f is one-one, but g need not be.

5. (9 marks) Use the Euclidean algorithm to find a $\gcd(x^4 + x^2 + x - 1, x^3 + 1)$ in $\mathbb{R}[x]$. Show the steps in the calculation.

6. (12 marks) On $\mathbb{N}$ with the partial order given by 'divides', define $\gcd(a, b, c)$ to be the greatest element of the set of natural numbers which are common divisors of the 3, natural numbers a, b and c. Write $e = \gcd(a, b, c)$ and $f = \gcd(d, c)$ where $d = \gcd(a, b)$. Prove that $e = f$.