NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date.** Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet. These are available from outside the Resource Centre. PLEASE SHOW ALL WORKING. Also if we believe you have COPIED someone else's script or that you have let someone else COPY YOUR SCRIPT, then you will get NO MARKS.

1. Let $a, b, c \in \mathbb{Z}$. Prove that if $a \mid b$ and $b \mid c$ then $a \mid c$.

2. Use the Euclidean algorithm to find the greatest common divisor of 2700 and 17,640. Show your working carefully.

3. Prove the theorem from lectures:
If $a, b, q, r \in \mathbb{Z}$ and $a = bq + r$ then $\gcd(a, b) = \gcd(b, r)$.

4. Let $a, b \in \mathbb{Z}$, not both zero, then the following statements are equivalent for a positive common divisor d of a and b:
(i) $c \leq d$ for all common divisors c of a and b.
(ii) $c \mid d$ for all common divisors c of a and b.

(a) That (ii)⇒(i) is true is *almost* immediate from some result in lectures. What is that result? Show something about d that enables us to use that result.

(b) Prove that (i)⇒(ii) is true *by the following steps.* (Throughout c is a common divisor of a and b.)
(a) Show that a is a common multiple of c and d.
(b) Show that b is a common multiple of c and d.
(c) Let $m = \text{lcm}(d, c)$. Explain why $m \mid a$ and $m \mid b$.
(d) Explain why we can use the last results to conclude that $m \leq d$.
(e) On the other hand it is obvious that $d \leq m$. Why?
(f) Use the last two results to deduce that $c \mid d$.