MATHS255FC	Assignment 4	Due: 4pm, Wednesday 10 April 2002

NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date.** Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet. These are available from outside the Resource Centre. PLEASE SHOW ALL WORKING. Also if we believe you have COPIED someone else's script or that you have let someone else COPY YOUR SCRIPT, then you will get NO MARKS.

1. Prove or give a counterexample: If $f : X \to Y$ is a function between the sets X and Y and B, C are subsets of Y then $f^{-1}(B \cup C) = f^{-1}(B) \cup f^{-1}(C)$.

2. Let
$$A = \{x \in \mathbb{R} \ : \ x = 1 - \frac{1}{n} \ \text{ for some } \ n \in \mathbb{N}\},$$

and let $B = A \cup \{1\}$. Define f to be the function $f : A \to B$ given by $f(x) = \frac{2x-1}{x}$ for $x \neq 0$, and $f(0) = 1$.

   (a) Show that f is a bijection. (Hint: First show that the formulae for f, applied to elements of A does yield elements of B as asserted.)

   (b) Show that if A and B are considered as posets by restricting the usual ordering on $\mathbb{R}$, then f is not an order isomorphism.

3. Prove by induction that, for every $n \in \mathbb{N}$, $11^n - 4^n = 7\ell$ for some $\ell \in \mathbb{N}$.