

NB: Please deposit your solutions in the appropriate box **by 4 p.m. on the due date.** Late assignments or assignments placed into incorrect boxes will not be marked. Use a mathematics department cover sheet. These are available from outside the Resource Centre. PLEASE SHOW ALL WORKING. Also if we believe you have COPIED someone else's script or that you have let someone else COPY YOUR SCRIPT, then you will get NO MARKS.

1. For each of the sentences in quotations, answer the following questions.
 - (i) Is it a statement? A predicate?
 - (ii) Are there free or bound variables? List these if so.
 - (iii) Where possible express each symbolically in terms of the statements, predicates, sets and relations given and using the notation of logic as in the lectures.
 - (iv) Where appropriate decide if it is a tautology, a contradiction or neither of these.
 - (a) "A and B are true or B is false." Here A and B are statements. [5 marks]
 - (b) "He is stronger than Helen Clark." [5 marks]
 - (c) "If either A is true or B is true then it is not the case that both A and B are true." Here A and B are statements. [5 marks]
 - (d) "If A then why not B?". Here A and B are statements. [5 marks]
 - (e) If $y + x^2 < 0$ then $y < 0$. [5 marks]
 - (f) "Joe never needs help from anybody" in terms of the set of all people 'P', the set of all times 'T', and the predicate $R(x, y, t) =$ 'person x needs help from person y at time t '. (Here Joe is a given particular person.) [5 marks]
 - (g) "Always everybody needs help from somebody" using the same notation as part f. [5 marks]
2. Assume A, B, and C are statements. Construct truth tables for each of the following pairs of compound statements. State (giving a reason) whether either implies the other or whether they are equivalent. [10 marks]
 - (a) $A \wedge (B \vee C)$, $(A \vee B) \wedge (A \vee C)$
 - (b) $\sim(A \vee \sim B)$ $\sim A \wedge B$
3. Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$ be the function given by $f(x) = 3x^2 + 2$.
 - (a) Use a carefully direct proof to show that if $f(n) < f(k)$ and $0 < k + n$ then $n < k$. [5 marks]
 - (b) Prove that the statement $(f(x) = f(y) \Rightarrow x = y)$ is false. Explain briefly how you will do this. [4 marks]