MATHS 255: Principles of Mathematics
CLASS TEST

3-4 pm THURSDAY 13th September 2001

NOTE: Attempt all questions. Write your answers in the examination booklet provided. You may refer to books, handouts and lecture notes, and calculators may be used, but communication with other students during the test is **not** permitted.

1. Let $A(n)$ be the predicate:

$$\text{If } n \text{ is not prime, then } \exists p \in \mathbf{N} \ \ p \text{ is prime and } p \mid n.$$

(a) [5] Write down the negation of $A(n)$,

(b) [10] Determine with reasons whether or not it is true that $\exists n \in \mathbf{N} \ \ A(n)$.

(c) [10] Determine with reasons whether or not it is true that $\forall n \in \mathbf{N} \ \ A(n)$.

2. A relation $\lhd$ is defined on $\mathbf{R} \times \mathbf{R}$ by $(a,b) \lhd (c,d)$ if and only if

$$\text{either } a < c \text{ or } (a = c \text{ and } b \le d).$$

($\lhd$ is sometimes called "dictionary ordering," and in fact it is a total ordering of $\mathbf{R} \times \mathbf{R}$.)

(a) [5] Put the following three elements of $\mathbf{R} \times \mathbf{R}$ in order:

$$(\pi, \sqrt{10}), \left(\frac{22}{7}, \pi\right), \left(\pi, \frac{22}{7}\right) \quad [\text{Hint: } \pi < \frac{22}{7} < \sqrt{10}].$$

(b) [10] Assuming that $\lhd$ is a total ordering of $\mathbf{R} \times \mathbf{R}$., sketch the "closed interval" $\{(a,b) : (1,2) \lhd (a,b) \lhd (2,1)\}$.

(c) [15] Assuming that $\le$ is a total ordering on $\mathbf{R}$, but without assuming that $\lhd$ is a total ordering on $\mathbf{R} \times \mathbf{R}$, prove that $\lhd$ is antisymmetric.

3. [20] Find and prove a formula for $\frac{1}{n} + \sum_{i=2}^{n} \frac{1}{i(i-1)}$ valid for all integers $n \ge 2$,

4. [25] Find a greatest common divisor $d(x)$ of the polynomials $a(x) = x^3 + x^2 + x + \bar{1}$, $b(x) = x^4 - x^3 + x + \bar{1}$ in $\mathbf{Z}_2[x]$, and find $u(x), v(x) \in \mathbf{Z}_2[x]$ such that $d(x) = a(x)u(x) + b(x)v(x)$.