MATHS 255 Class Notes
Chapter 4 Cont'd

Proposition (4.2.11) A poset A has at most one largest element.

Proposition (4.2.19) A non-empty subset K of a poset A has at most one lub.

A poset A has the *lub property* if every nonempty subset of A that is bounded above has a lub.

Theorem (4.2.23). If A is a poset (*note misprint in text*) which has the lub property, then A has the glb property (i.e. every nonempty subset that is bounded below has a glb.)

Equivalence Relations

Note: We omit 4.3.4 – 4.3.12 on pages 44-46, and approach the ideas somewhat more simply.

An *equivalence relation on* S is a relation on S which is

- reflexive
- symmetric
- transitive

Examples:

1. S = all $n \times n$ matrices. $A \rho B$ if A is similar to B. (that is,

   $\exists P \in S,\ P$ is invertible and $P^{-1}AP = B$.

2. S = all $m \times n$ matrices. $A \rho B$ if A is row equivalent to B. (that is,

   $\exists P \in S,\ P$ is invertible and $PA = B$.

3. $S = \mathbf{R}$ and $x \rho y \Leftrightarrow \lfloor x \rfloor = \lfloor y \rfloor$ (Here, $\lfloor x \rfloor$ = largest integer n such that $n \le x$.)

4. S = all people,

   $x \rho y \Leftrightarrow x, y$ have the same mother.

Equivalence classes

If S is a set and ρ is an equivalence relation on S, and $x \in S$, then the *equivalence class of* x is

$$\{y \in S: \ x\rho y\},$$

or in other words, the set of all "relatives of *x*."

Example: cf.3. above: the equivalence class of π is $[3, 4)$, and so is the equivalence class of 22/7.

Every element of S is in one and only one equivalence class.

Partitions

A *partition of* S is a collection of subsets of S such that every element of S belongs to one and only one of the subsets.

In other words, the subsets (called "parts of S") are non-overlapping and exhaustive.

Examples

1. A jigsaw puzzle is a partition of a picture.

2. $\{[n, n+1): \ n \in \mathbf{Z}\}$ is a partition of **R**..

Connection between equivalence relations and partitions

There is a direct correspondence between partitions of a set and equivalence relations on the set:

> Given a partition, define an equivalence relation by saying that two elements are equivalent iff they lie in the same part.
>
> Given an equivalence relation, define a partition by saying that the parts are the equivalence classes.