MATHS 255 Class Notes
§2.5 Cont'd and §2.6

Converse The *converse* of $A \Rightarrow B$ is $B \Rightarrow A$.

e.g. Statement: "All dogs have four legs."
Converse: "All creatures with four legs are dogs."

e.g. Predicate: $X \subseteq Y$. Converse: $Y \subseteq X$.

A statement is generally **not** equivalent to its converse and the two should not be confused.

Contrapositive The *contrapositive* of $A \Rightarrow B$ is $\sim B \Rightarrow \sim A$.

e.g. Statement: "All dogs have four legs."
Contrapositive: "All creatures not having four legs are not dogs."

Any statement and its contrapositive are equivalent.

e.g. Prove by contrapositive that for any integers x,y, if xy is even then x is even or y is even.

§2.7 Negation

Important equivalences:

1. $\sim (A \wedge B) \Leftrightarrow (\sim A \vee \sim B)$

2. $\sim (A \vee B) \Leftrightarrow (\sim A \wedge \sim B)$

3. $(A \Rightarrow B) \Leftrightarrow (\sim A \vee B)$

e.g. (p.4) Negate this: "If people recognize you every time you go to the store, then either you live in a small town or else you are a famous celebrity."

Negating quantified statements:

4. $\sim (\forall x \;\; P(x)) \;\Leftrightarrow\; \exists x \;\; \sim P(x)$

5. $\sim (\exists x \;\; P(x)) \;\Leftrightarrow\; \forall x \;\; \sim P(x)$

e.g. (p.5) Negate this: "For every poison there is a chemical that is the antidote."

Proof by contradiction = **Indirect proof**

Proving that $A \Rightarrow B$ is true (i.e. that it is a tautology) is equivalent to proving that $\sim (A \Rightarrow B)$ is false (i.e. that it is a contradiction). From 2. and 3 above, we have that

6. $\sim (A \Rightarrow B) \;\Leftrightarrow\; (A \wedge \sim B)$

e.g. Negate this (p,19): "If Martha is sick, then Joe misses work."

Proving that $A \Rightarrow B$ is true is therefore equivalent to proving that $A \wedge \sim B$ is a contradiction. This is a *proof by contradiction*.

e.g. Prove indirectly that for any integers x,y, if xy is even then x is even or y is even.

Proof by contradiction is closely related to proof by contrapositive.