MATHS 255 Class Notes
Chapter 6 Induction Cont'd

Variations of PMI

1. Different starting point

If r is an integer and $P_r,\ P_{r+1},\ P_{r+2},\ \cdots$ are statements satisfying

1. P_r is true,
2. For all integers $k = r,\ r+1,\ r+2,\ \cdots,$ $P_k \Rightarrow P_{k+1}$ is true,

then $\forall n \geq r,\ P_n$ is true.

Example

Find all $n \geq 0$ such that $n! > 3^n$.

2. Complete Induction ("Strong Form")

If r is an integer and (P_n) is a sequence of statements satisfying

1. P_1 is true,
2. For all $k \in \mathbf{N},\ P_1,\ P_2,\ \cdots, P_k$ together imply P_{k+1} is true,

then $\forall n \geq r,\ P_n$ is true.

Example

Every integer ≥ 2 is a prime or a product of primes.

3. Well-ordering

A totally ordered set $S, \leq$ is *well-ordered* if every non-empty subset of S has a least element.

Theorem (7.1.3) $\mathbf{N}$ is well-ordered.