MATHS 255 Class Notes
Chapter 5. Functions, Continued

Extensions of function notation (5.2 contd)

Suppose $f\colon X \to Y$ is a function (not necessarily invertible).

If $A \subseteq X$, we define (cf. 5.2.13)
$f(A) = \{f(a)\colon\ a \in A\}$, the *image of A* under *f*.

If $B \subseteq Y$, we define (cf. 5.2.8)
$f^{-1}(B) = \{a \in A\colon\ f(a) \in B\}$, the *pre-image* of B under *f*.

The same notations f and f^{-1} are used for these functions from $\wp(A) \to \wp(B)$ and from $\wp(B) \to \wp(A)$ respectively.

Again, notice that we use f^{-1} even when f has no inverse function, and we rely on the context to make clear what is meant by f^{-1}.

For all subsets A_1, A_2 of A and B_1, B_2 of B

1. $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.
2. $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$.
3. $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$.
4. $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.

Order preserving functions on posets (5.3)

If $X, \prec$ and $Y, \lhd$ are posets, and $f\colon X \to Y$ is a function such that

$$\forall a, b \in X \;\; a \prec b \Leftrightarrow f(a) \lhd f(b)\,],$$

then f is *order preserving.*

If in addition f is one-to-one and onto, then f is an *order isomorphism*.

Order isomorphic posets have equivalent order structures. Order isomorphisms preserve all such things as

order,
maximal and minimal elements
upper and lower bounds
totally ordered subsets
immediate successors and predecessors

Sequences

A *sequence of elements of A* is a function $s\colon \mathbf{N} \to A$

$s(n)$ is sometimes denoted s_n, and s is sometimes denoted (s_n).

Recursively defined sequences: A formula for s_n is given based on some previous terms.

Examples:

Fibonacci sequence

Factorial

Iterated map. $f\colon A \to A$, $a_0 \in A$, $a_i = f(a_{i-1})$.

Sequence terminology

Distinct terms

Constant sequence

In posets:

increasing, strictly increasing
decreasing, strictly decreasing
monotonic

bounded above, below
upper bound, lower
bounded

subsequence

Binary Operations (5.5)

If A is a set, then a *binary operation on* A is any function $* : A \times A \to A$.

Properties of Binary operations:

Commutative.

Associative.