DEPARTMENT OF MATHEMATICS

MATHS 255FC	Assignment 9	Due 3rd October 2001

1. (a) Consider the sequence $\left\{\frac{5n+7}{7n+5},\ n \in \mathbf{N}\right\}$.

(i) Show the sequence is strictly decreasing and bounded below.
(ii) Use first principles ($\varepsilon - N$ proof) to find and prove its limit.

(b) Use (a) to show the sequence $\left\{\left(\frac{5n+7}{7n+5}\right)^2,\ n \in \mathbf{N}\right\}$ is convergent and find its limit.

2. Consider the following iterative sequence defined by the following: (i) $x_1 = 1$, (ii) $x_{n+1} = \sqrt[3]{k^2 x_n}$, $k > 1$.

(a) Find an upper bound for the sequence.
(b) By examining the ratio of successive terms, show that the sequence is increasing.
(c) Use the recursion relation (ii) to find a hypothetical limit *l*.
(d) Hence find the limit of the sequence.

3. If $\{x_n\}$ is bounded, consider the sequence $\{g_n = \text{glb}\{x_k : k \geq n\}\}$

(a) Show $\{g_n\}$ is bounded.
(b) Show that if $\{x_n\}$ is convergent then $\{g_n\}$ is also and $\{x_n\}$ and $\{g_n\}$ have the same limit.

4. Let $X = (0,1]$

(a) If $e: X \times X \to \mathbf{R}$ defined by $e(x,y) = \left|\frac{1}{x} - \frac{1}{y}\right|$ show (X,e) is a metric space.

(b) Two metric spaces (X,d) and (Y,e) are equivalent if and only if there is a 1-1 onto function $h: X \to Y$ such that $d(x,y) = e(h(x),h(y))$. If (X,d) is defined to be the standard metric i.e. $d(x,y) = |x-y|$ show that (X,d) and (X,e) are not equivalent.