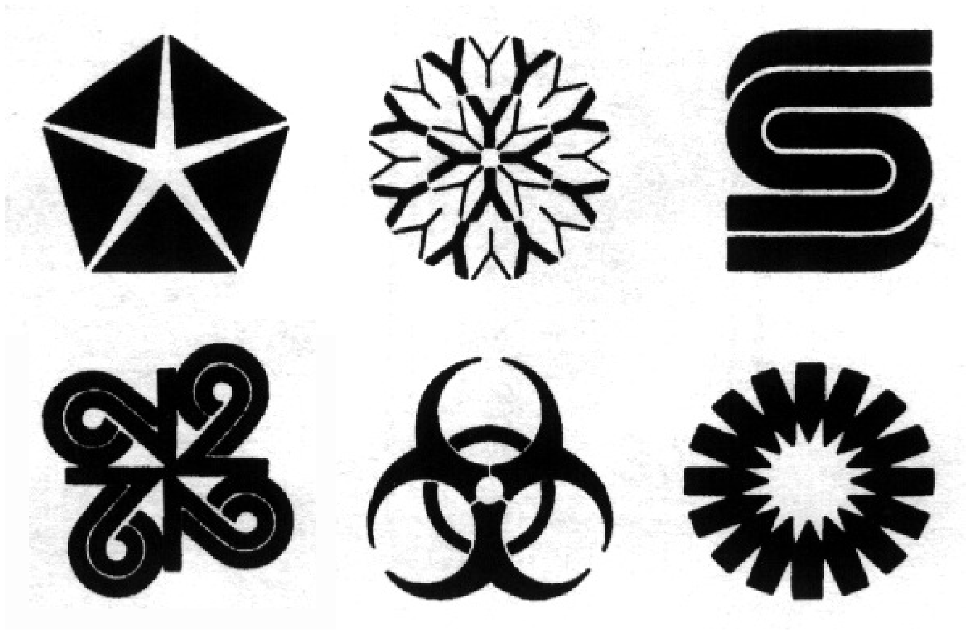
DEPARTMENT OF MATHEMATICS

MATHS 255FC | Assignment 7 | Due 19th Sept 2001

1. For each design below determine the symmetry group.



2. If G is a group, Show that $(a*b)^2 = a^2 * b^2 \Rightarrow a*b = b*a$.

3. If G is a group for which every element $g \in G$ has $g^2 = e$, Show that G is commutative.

4. Let G be the group of matrices of each of the linear transformations corresponding to the group of symmetries of the square with vertices (±1,±1) under matrix multiplication. Show $G$is isomorphic to D_4.

5. (a) Show that $\mathbf{Z}_9 \setminus \{0\}$ is not a multiplicative group.

(b) Let $U(9) = \{ \bar{n} : n \textit{ is relatively prime to } 9 \}$.
Show $(U(9), \bullet_9)$ is a multiplicative group, where $\bullet_9$ is multiplication modulo 9.

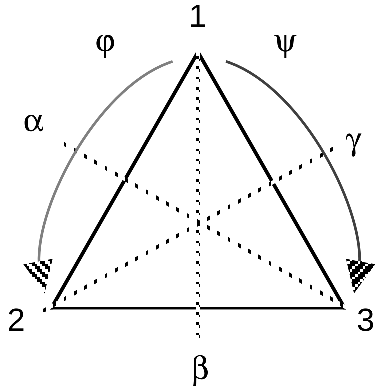
(c) Show *U(9)* is isomorphic to the additive group $\mathbf{Z}_6$.

3. Consider the symmetries of an equilateral triangle.

(i) Show this group is isomorphic to the group of permutations of three elements S_3.

The Cayley table for the group illustrated on page 5 of the Groups notes is shown below reordered

	e	φ	ψ	α	β	γ
e	e	φ	ψ	α	β	γ
φ	φ	ψ	e	γ	α	β
ψ	ψ	e	φ	β	γ	α
α	α	β	γ	e	φ	ψ
β	β	γ	α	ψ	e	φ
γ	γ	α	β	φ	ψ	e



The isomorphism between S_3 and the group D_3 of symmetries of the triangle is defined by each of α, β, γ with the reflections in the three axes shown in the diagram. φ, ψ are associated with rotations of $\frac{2\pi}{3}$ and $\frac{4\pi}{3} = \frac{-2\pi}{3}$ respectively as shown in the triangle diagram. This is clearly an isomorphism because the symmetries permute the vertices exactly as in S_3. **[6]**

(ii) Show the subgroup consisting of rotations of the triangle is isomorphic with the additive group $\mathbf{Z}_3$ of integers modulo 3.

The first three elements in the above Cayley table i.e. the identity and the two rotations are clearly additive angles of $0, \frac{\pi}{3}, \frac{2\pi}{3}$ modulo 2π precisely the same as the additive group of 0, 1, 2 mod 3. viz

0	1	2
1	2	0
2	0	1

[2]

The Cayley table of *D_4*

	R_0	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_0	R_0	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_{90}	R_{90}	R_{180}	R_{270}	R_0	D'	D	H	V
R_{180}	R_{180}	R_{270}	R_0	R_{90}	V	H	D'	D
R_{270}	R_{270}	R_0	R_{90}	R_{180}	D	D'	V	H
H	H	D	V	D'	R_0	R_{180}	R_{90}	R_{270}
V	V	D'	H	D	R_{180}	R_0	R_{270}	R_{90}
D	D	V	D'	H	R_{270}	R_{90}	R_0	R_{180}
D'	D'	H	D	V	R_{90}	R_{270}	R_{180}	R_0