DEPARTMENT OF MATHEMATICS

MATHS 255	Assignment 6	Due: 12 September, 2001

Note: Please deposit your answers in the appropriate box outside the Student Resource Centre in the basement of the Mathematics/Physics building **by 4 pm on the due date.** Late assignments will not be marked. Use a Mathematics Department cover sheet which is available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. Prove that any natural number n is congruent modulo 9 to the sum of its decimal digits. [We say that $a_m a_{m-1} \cdots a_0$ is the decimal expansion of n if

$$\forall i,\ a_i \in \mathbf{Z},\ \ 0 \le a_i \le 9\ ;\ \ a_m \ne 0 \quad \text{and } n = \sum_{i=0}^{m} a_i (10)^i .$$

You may assume the existence and uniqueness of such an expansion for each $n \in \mathbf{N}$.]

2. Find all invertible elements of $\mathbf{Z}_{40}$, and find the inverse of each of them. [Use a calculator or computer if you wish, but explain your methods.]

3. Find the quotient and remainder when $a(x)$ is divided by $b(x)$ where

$$a(x) = x^7 + x^5 - x^4 + x^3 + x^2 - x + \overline{1} \quad \text{and} \quad b(x) = x^3 - x + \overline{1},$$

(a) assuming $a(x)$, $b(x)$ are in $\mathbf{Z}_2[x]$.

(b) assuming $a(x)$, $b(x)$ are in $\mathbf{Z}_3[x]$.

4. Write down <u>all</u> polynomials of degree ≤ 3 in $\mathbf{Z}_2[x]$, and indicate those which are irreducible. Also write down all <u>irreducible</u> polynomials of degree 4 in $\mathbf{Z}_2[x]$. Explain your answers.