DEPARTMENT OF MATHEMATICS

MATHS 255	Assignment 4	Due: 15 August, 2001

Note: Please deposit your answers in the appropriate box outside the Student Resource Centre in the basement of the Mathematics/Physics building **by 4 pm on the due date.** Late assignments will not be marked. Use a Mathematics Department cover sheet which is available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. (a) Prove or give a counterexample: If $f: X \to Y$ is a function and C,D are disjoint subsets of Y then $f^{-1}(C)$, $f^{-1}(D)$ are disjoint subsets of X.

(b) Prove or give a counterexample: If $f: X \to Y$ is a function and A,B are disjoint subsets of X then $f(A)$, $f(B)$ are disjoint subsets of Y.

2. Show that if u is a subsequence of t and t is a subsequence of s, then u is a subsequence of s. [Use the definition of subsequence in the text, and prove what you need to prove about composition of increasing functions.]

3. Determine which of the following are binary operations, and for those which are, determine whether they are associative, commutative.

(a) Symmetric difference on the set of all *finite* subsets of an *infinite* set A.

(b) $*$ on $\mathbf{R} \setminus \{1\}$ defined by $a * b = a + b - ab$ for all $a, b \in \mathbf{R} \setminus \{1\}$.

4. (a) Prove by induction that for each $n \in \mathbf{N}$, $4^n - 1$ is divisible by 3.

(b) Criticise the following:

Theorem We are given n coins, where n is an integer > 1. All but one of the coins are the same weight and the other is heavier. We have a balance. Then 4 weighings suffice to discover which coin is heavier.

Proof
By induction.
When n=2 the result is clear. Suppose we have proved the result for k coins. We are now given k+1 coins. We proceed as follows. Set one coin aside. Apply the procedure for k coins to the remaining k coins. If we find the heavy coin then we are finished. If not, then the heavy coin is the one we set aside. Thus we have a procedure for k+1 coins. The theorem follows by induction.