DEPARTMENT OF MATHEMATICS

MATHS 255	Assignment 3	Due: 8 August, 2001

Note: Please deposit your answers in the appropriate box outside the Student Resource Centre in the basement of the Mathematics/Physics building **by 4 pm on the due date.** Late assignments will not be marked. Use a Mathematics Department cover sheet which is available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. Suppose that X is a poset with partial ordering $\leq$, and suppose that A is a non-empty subset of X. Show that if A has a least upper bound and a greatest lower bound, then $\mathrm{glb}(A) \leq \mathrm{lub}(A)$.

2. Let $A = \mathbf{N} \times \mathbf{N}$, and define a relation ~ on A by $(a,b) \sim (c,d) \iff b + c = a + d$. Prove that ~ is an equivalence relation on A, and describe the equivalence classes.

   [Note: This set of equivalence classes, endowed with appropriate definitions of addition and multiplication, is sometimes called the set of *integers*.]

3. (a) Prove that if functions $f: A \to B$ and $g: B \to C$ are onto, then so is $g \circ f$.

   (b) Prove that if $f: A \to B$ is a function and the inverse relation f^{-1} from B to A is a function, then f is one-to-one and onto.

4. Let $f: A \to B$ be a function, and define a new function $F: \wp(B) \to \wp(A)$ by

$$F(C) = \{a \in A \ : \ f(a) \in C\}$$

for each $C \subseteq B$. Prove that f is one-to-one if and only if F is onto.