DEPARTMENT OF MATHEMATICS

MATHS 255 Assignment 2 Due: 1 August, 2001

Note: Please deposit your answers in the appropriate box outside the Student Resource Centre in the basement of the Mathematics/Physics building **by 4 pm on the due date.** Late assignments will not be marked. Use a Mathematics Department cover sheet which is available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. Let A,B,C be sets. Prove that $(A \cap B) \cup C = A \cap (B \cup C) \iff C \subseteq A$.

Given sets A,B we define a set operation Δ (called "symmetric difference') as follows:

$$A \Delta B = (A \setminus B) \cup (B \setminus A).$$

2. If $A_1 = \{\varnothing\}$, $A_2 = A_1 \cup \{A_1\}$, $A_3 = A_2 \cup \{A_2\}$, then list the elements of A_2, A_3, and $A_2 \Delta A_3$.

3. Find a way to write $A \cup B$ and $A \setminus B$ using only the set operations $\cap$ and Δ.

[Hint: If $X \cap Y = \varnothing$ then $X \cup Y = X \Delta Y$.]

4. Indicate whether each of the following relations on the given set is reflexive, symmetric, antisymmetric, transitive. Explain each answer.

i. $A = \{p: p$ is a guest at a particular party$\}$. $x \rho y$ iff x knows the name of y.

ii. $A = \{p: p$ is a person in New Zealand$\}$. $x \rho y$ iff x is at least as tall as y.

iii. $A = P(\mathbf{N})$. $x \rho y$ iff $x \subseteq y$.

iv. $A = \{a,b,c\}$ (distinct elements). $\rho = \{(a,a),(a,b),(b,c),(a,c),(c,a)\}$.

v. $A = \{ l: l$ is a line in the Cartesian plane$\}$. $x \rho y$ iff x and y are either parallel or identical lines.