DEPARTMENT OF MATHEMATICS

| MATHS 255FC | Assignment 10 | Due 10th October 2001 |
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**1.** Consider $f(x)=\begin{cases} x.\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$

(a) Show from first principles that *f(x)* is continuous at *x* = 0.
(b) Is *f(x)* continuous at other points in **R**? Explain.
(c) Determine whether or not *f(x)* is differentiable at 0?
(d) Is *f(x)* differentiable at other points in **R**? Explain.

**2.** Determine where the following function is continuous and where it is differentiable:

$$f(x)=\begin{cases} x^4-3x^3+2x^2, & x\in \mathbf{Q} \\ 0, & x\notin \mathbf{Q} \end{cases}$$

**3.** (a) Suppose that $f$ is continuous and bounded on **R**.
Either prove that $f$ attains a maximum or a minimum value on **R**, or give a counter example.

(b) Suppose now that $f$ is continuous on **R** and $f(x)\to 0$ as $x\to\pm\infty$
Does this change your conclusion? Prove your result or give a counter-example.

**4.** Let a real valued function $f$ be continuous on the closed interval $[a,b]$. Suppose that for each $x\in[a,b]$ there exists there exists a $y\in[a,b]$ such that $|f(y)|\leq\frac{1}{2}|f(x)|$. Prove there exists a $z\in[a,b]$ for which $f(z)=0$. (Hint: Use the Extreme Value Theorem).