DEPARTMENT OF MATHEMATICS

MATHS 255	Assignment 1	Due: 25 July. 2001

Note: Please deposit your answers in the appropriate box outside the Student Resource Centre in the basement of the Mathematics/Physics building **by 4 pm on the due date.** Late assignments will not be marked. Use a Mathematics Department cover sheet which is available from outside the Resource Centre. PLEASE SHOW ALL WORKING.

1. Write the following in symbolic form, using symbols $\sim, \wedge, \vee, \Rightarrow, \Leftrightarrow, \forall, \exists$, letters x,y,z ... for free variables and capital letters A,B,C... for statements.

   a. Either Joe is smart or he is lucky but not both.

   b. Doing homework regularly is a necessary condition for me to pass this course, but it is not sufficient.

   c. It is not the case that if you are either unkind to me or unkind to my friend then I will neither sing to you nor talk to you. (Also rewrite this in a more positive way.)

   d. Every real number is a sum of two distinct real numbers.

   e. Given any two real numbers, there is a real number which is less than their sum.

2. Assume A,B,C are statements. Construct truth tables for the following pairs of statements. State whether either implies the other and whether or not they are equivalent:

   a. $A \wedge (B \vee C)$, $(A \wedge B) \vee (A \wedge C)$.

   b. $A \wedge \sim B$, $\sim(\sim A \vee B)$.

   c. $(A \Leftrightarrow B) \wedge (B \Rightarrow \sim C) \wedge C$, $\sim A$.

3. Let $f: N \to N$ be given by $f(x) = x^3 + 5x$. (N = natural numbers)

   a. Use a direct proof to show that if $n < k$ then $f(n) < f(k)$.

   b. Use a proof by contrapositive to show that if $f(n) < f(k)$ then $n < k$.

   c. Use a proof by contradiction to show that if $f(n) = f(k)$ then $n = k$.

   d. Use a proof by cases to prove that $f(n)$ is a multiple of 3 for all $n \in N$. [You may assume that every integer can be written uniquely in the form $3k$, $3k + 1$, or $3k + 2$.]

4. a. Prove that for any sets X, Y, $X \subseteq Y$ if and only if $P(X) \subseteq P(Y)$.

   b. Prove that for any sets A, B, $(A \setminus B) \cap B = \varnothing$ and $(A \setminus B) \cup B = A \cup B$.