THE UNIVERSITY OF AUCKLAND **MATHS 255 FC**

CLASS TEST, FIRST SEMESTER, 2001

MATHEMATICS

Principles of Mathematics

(Time allowed: 90 MINUTES)

1. For each natural number n, let $A(n)$ be the statement

"If n is prime then $n+2$ is prime."

(a) Write down the contrapositive of $A(n)$.

(b) Write down the converse of $A(n)$.

(c) Write down the negation of $A(n)$.

(d) Is $A(n)$ true for some $n \in \mathbb{N}$? If so, give an example, if not give a proof.

(e) Is $A(n)$ true for every $n \in \mathbb{N}$? If so, give a proof, if not give a counterexample.

(f) Is the contrapositive of $A(n)$ true for some $n \in \mathbb{N}$? Is it true for all $n \in \mathbb{N}$? Give brief reasons for your answer.

(g) Is the converse of $A(n)$ true for some $n \in \mathbb{N}$? Is it true for all $n \in \mathbb{N}$? Give brief reasons for your answer.

2. Let X be a set, and $f : X \to \mathbb{R}$ a function. Define a relation ρ on X by declaring that, for $x, y \in X$,

$$x \;\rho\; y \qquad \text{if and only if} \qquad f(x) \leq f(y).$$

(a) Show that ρ is reflexive and transitive.

(b) Show that ρ is antisymmetric if and only if f is one-to-one.

3. A *blah* function is a function $f : \mathbb{Z} \to \mathbb{R} \setminus \{0\}$ such that, for every $x, y \in \mathbb{Z}$, $f(x+y) = f(x) \cdot f(y)$.

(a) Show that if f is a blah function then $f(0) = 1$ [Hint: $f(0+0) = f(0)$, and the only solutions of $x^2 = x$ in $\mathbb{R}$ are 0 and 1.]

(b) Show that if f is a blah function and $x \in \mathbb{Z}$ then $f(-x) = \dfrac{1}{f(x)}$.

(c) Let f and g be blah functions with $f(1) = g(1)$. Use induction to show that $f(x) = g(x)$ for all $x \in \mathbb{N}$.

(d) Let f and g be blah functions with $f(1) = g(1)$. Show that $f = g$.

4. (a) Let $a, b, n \in \mathbb{N}$. Suppose that there exists an integer c such that $ac \equiv 1 \pmod{n}$. Show that the equation $\overline{a} \cdot_n \overline{x} = \overline{b}$ has a unique solution $\overline{x} \in \mathbb{Z}_n$.

(b) Find all integer solutions x and y to the equation $35x + 16y = 3$.