1. (a) "17 is an even number" is a statement.

(b) "n is a prime number" is a predicate (with n as a free variable).

(c) "If n is a prime number then n is odd" is a predicate (with n as a free variable).

(d) "Is 13 a prime number?" is neither a statement nor a predicate.

(e) "Every even number is the sum of two odd numbers" is a statement.

2. (a) We have the following truth table:

A	B	$A \implies B$	$\sim B$	$A \implies \sim B$	$(A \implies B) \vee (A \implies \sim B)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

Thus $(A \implies B) \vee (A \implies \sim B)$ is a tautology.

(b) We have the following truth table:

A	B	$A \implies B$	$\sim A$	$\sim A \implies B$	$(A \implies B) \vee (\sim A \implies B)$
T	T	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	T	F	T

Thus $(A \implies B) \vee (\sim A \implies B)$ is a tautology.

(c) We have the following truth table:

A	B	$A \implies B$	$\sim A$	$\sim B$	$\sim A \implies \sim B$	$(A \implies B) \wedge (\sim A \implies \sim B)$
T	T	T	F	F	T	T
T	F	F	F	T	T	F
F	T	T	T	F	F	F
F	F	T	T	T	T	T

Thus $(A \implies B) \wedge (\sim A \implies \sim B)$ is neither a tautology nor a contradiction.

(d) We have the following truth table:

A	B	$A \implies B$	$\sim(A \implies B)$	$A \wedge \sim(A \implies B)$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

Thus $A \wedge \sim(A \implies B)$ is neither a tautology nor a contradiction.

3. (a) The contrapositive of $A(n)$ is "If $n+1$ is prime then n is not prime".

(b) The converse of $A(n)$ is "If $n+1$ is not prime then n is prime".

(c) The negation of $A(n)$ is "n is prime and $n+1$ is prime".

(d) Yes, $A(n)$ is true for some n: for example, $A(5)$ is true.

(e) No, $A(n)$ is not true for every natural number n: as a counterexample, $A(2)$ is false.

(f) Since the contrapositive is equivalent to $A(n)$ itself, from (d) and (e) we see that the contrapositive is true for some $n \in \mathbb{N}$ but not for all $n \in \mathbb{N}$.

(g) The converse is true for some but not all $n \in \mathbb{N}$: for example the converse of $A(5)$ is true, while the converse of $A(8)$ is false.

4. (a) Suppose n is even. Then $n = 2k$ for some natural number k, so

$$\begin{aligned} f(n) &= (2k)^2 + 2(2k) \\ &= 2(2k^2 + 2k), \end{aligned}$$

and $2k^2 + 2k \in \mathbb{N}$, so $f(n)$ is even.

(b) Suppose n is not even. Then n is odd, so $n = 2k+1$ for some $k \in \mathbb{Z}$. But then

$$\begin{aligned} f(n) &= (2k+1)^2 + 2(2k+1) \\ &= 4k^2 + 4k + 1 + 4k + 2 \\ &= 2(2k^2 + 4k + 1) + 1, \end{aligned}$$

and $(2k^2 + 4k + 1) \in \mathbb{Z}$, so $f(n)$ is odd, so $f(n)$ is not even. Hence, by contraposition, if $f(n)$ is even then n is even.

(c) Suppose, for a contradiction, that $f(n+k)$ is odd and (n is odd or k is odd) is false, in other words n and k are both even. But then $n+k$ is even, so by part (a) we have $f(n+k)$ is even, a contradiction. Hence, by contradiction, if $f(n+k)$ is odd then n is odd or k is odd.