DEPARTMENT OF MATHEMATICS

MATHS 255FC	Assignment 9	Due 9th May 2001

1. Prove each of the following specifically from the axioms given in the handout on real numbers.

(a) Given $a, b \in \mathbf{R}$, show there is a unique *x* such that $ax = b$.
If *x=b/a* is defined to be $ax = b$, show:
(i) $a/b + c/d = (ad + bc)/bd$ if $b, d \neq 0$. (ii) $(a/b).(c/d) = ac/bd$ if $b, d \neq 0$.
(b) (i) $x < y,\ y < z \Rightarrow x < z$ (ii) $x < y,\ z > 0 \Rightarrow xz < yz$.
(c) (i) $|xy| = |x|.|y|$ (ii) $||x| - |y|| \leq |x - y|$.
(d) $A, B \subseteq \mathbf{R},\ A, B \neq \varnothing,\ A \subseteq B$ and *B* is bounded below, show glb$A \geq$ glb$B$.

2. Find the least upper bound and greatest lower bound of each of the following subsets of **R** if they exist and determine if either of these is an element of the set concerned.
(i) $\varnothing$ (ii) $[-1, \infty)$ (iii) $(-1, \sqrt{2}]$ (iv) $(-1, \sqrt{2}] \cap \mathbf{Q}$ (v) $\left\{ \left(1 + \frac{2}{n}\right)^n n = 1, 2, \ \ldots \right\}$

3. (a) A subset $S \subseteq X$ is dense in a metric space (X, d) if $\forall \varepsilon > 0,\ \forall x \in X\ \exists s \in S : |x - s| < \varepsilon$.
Show **Q** is dense in **R.**

(b) Show that $\forall x, y \in \mathbf{R} : x < y\ \exists\, q \in \mathbf{Q} : x < q < y$.

(c) Can you do the same as in (b) for $q \notin \mathbf{Q}$?

4. Show that any real valued function on the reals is continuous if and only if $x_i \to x_0 \Rightarrow f(x_i) \to f(x_0)$.