DEPARTMENT OF MATHEMATICS

MATHS 255FC	Assignment 8	Due 9th May 2001

1. (a) Show first principles that the sequence $\left\{\frac{4n+1}{n+4},\ n=0,\ 1,\ 2,\ ...\right\}$ is convergent.

(b) Hence or otherwise show that the sequence $\left\{\sqrt{\frac{4n+1}{n+4}},\ n=0,\ 1,\ 2,...\right\}$ is convergent.

2. Consider the set Σ_2 consisting of all sequences on the two elements 0 and 1, with the distance function between any two sequences $s = s_0, s_1, s_2, ...$ and $t = t_0, t_1, t_2, ...$ defined by $d(s,t) = \sum_{i=0}^{\infty} \frac{|s_i - t_i|}{2^{i+1}}$.

(a) Show (Σ_2, d) is a metric space i.e. that d is a metric distance function obeying **9.5.9** on p 123 of "Chapter 0".

(b) Show that

$$d(s,t) < \frac{1}{2^n} \Rightarrow s_i = t_i,\ i = 0,\ ...\ , n-1,$$
$$s_i = t_i,\ i = 0,\ ...\ , n-1 \Rightarrow d(s,t) \le \frac{1}{2^n}.$$

(c) Show that (Σ_2, d) is complete i.e. that every Cauchy sequence is convergent.

Hint: Use (b) to inductively define a sequence in Σ_2 to which your Cauchy sequence of sequences in Σ_2 converges.

3. A metric space is called compact if every sequence in the space has a convergent subsequence. A subset of **R** is compact if and only if it is closed an bounded.

(a) Use the fact that every bounded sequence has a convergent subsequence (see sequence notes for a proof of this) to show that [0,1] is compact.

(b) Give examples of a sequences in (0,1) and **R** which have no convergent subsequence in their respective sets.

(c) Show that (Σ_2, d) is compact.

Hint: If the chosen sequence of sequences in Σ_2 has only a finite number of sequences with 0 in the first position, choose the subsequence consisting of sequences with 1 in the first position. Otherwise pick the subsequence consisting of sequences with 0 in the first position. Repeat the process inductively.