

1. Determine whether the following sequences have distinct terms, are increasing, decreasing, monotonic or have an upper or lower bound. If the sequence has an upper bound find the least upper bound.

(i) $u_n = \frac{(-2)^n}{3^n(n+1)}, n = 0, 1, 2, \dots$

(ii) $u_n = n(2-n), n = 0, 1, 2, \dots$

(iii) $u_n = 1 - e^{-n}, n = 0, 1, 2, \dots$

2. Show that the following sets of numbers form (infinite) groups with respect to ordinary multiplication:

(i) $\{2^k : k = 0, \pm 1, \pm 2, \dots\}$

(ii) $\left\{ \frac{1+2m}{1+2n} : m, n = 0, \pm 1, \pm 2, \dots \right\}$

3. Consider the symmetries of an equilateral triangle.

(i) Show this group is isomorphic to the group of permutations of three elements S_3 .

(ii) Show the subgroup consisting of rotations of the triangle is isomorphic with the additive group \mathbf{Z}_3 of integers modulo 3.

4. Show that \mathbf{Z}_6 with the zero deleted is not a multiplicative group, but that \mathbf{Z}_7 with the zero deleted is a multiplicative group isomorphic to the additive group \mathbf{Z}_6 .