1. Let $f : A \to B$ and $g : B \to C$ be functions.

(a) Suppose that $g \circ f$ is onto and g is one-to-one. Show that f is onto.

(b) Give an example to show that we cannot omit the hypothesis that g is one-to-one in the previous part.

2. A function $f : \mathbb{R} \to \mathbb{R}$ is said to be *order-preserving* if, for all $x, y \in \mathbb{R}$,

$$x \leq y \qquad \text{iff} \qquad f(x) \leq f(y).$$

Show that if f is order-preserving then it is one-to-one. Does the converse hold?

3. A function $f : \mathbb{R} \to \mathbb{R}$ is said to be a *foo* function if $f(x+1) = f(x) + 1$ for every $x \in \mathbb{R}$, and a *bar* function if $f(x+1) = f(x) - 1$ for all $x \in \mathbb{R}$.

(a) Show that if f and g are both foo functions then $f \circ g$ is a foo function.

(b) Prove that $f : \mathbb{R} \to \mathbb{R}$ is a foo function if and only if for every $x \in \mathbb{R}$ we have $f(x-1) = f(x)-1$.

(c) Show that if f is a foo function and g is a bar function then $f \circ g$ is a bar function.

4. Prove by induction that, for every $n \in \mathbb{N}$, $n^3 + 5n$ is a multiple of 6. [You may assume that $n^2 + n$ is even for all $n \in \mathbb{N}$.]