1. Let $(A, \leq)$ be a poset. Define a relation $\preccurlyeq$ on $A^2 = A \times A$ by declaring that

$$(a, b) \preccurlyeq (c, d) \qquad \text{iff} \qquad (a \leq c \wedge b \leq d).$$

Show that $\preccurlyeq$ is a partial order on A^2.

2. Let $A \subseteq \mathbb{R}$, and let l be an upper bound for A. Show that l is least upper bound for A iff for every $\varepsilon \in \mathbb{R}$ with $\varepsilon > 0$ we have $(l - \varepsilon, l] \cap A \neq \varnothing$. [Hint: think about what it means for $l - \varepsilon$ to fail to be an upper bound for A.]

3. Let $A = \mathbb{Z} \times \mathbb{N}$. Define a relation $\sim$ on A by declaring that

$$(a, b) \sim (c, d) \qquad \text{iff} \qquad ad = bc.$$

Show that $\sim$ is an equivalence relation on A.

4. Let $f : A \to B$ be a function. Define a new function $F : \mathcal{P}(B) \to \mathcal{P}(A)$ by declaring that, for $C \subseteq B$,

$$F(C) = \{\, a \in A : f(a) \in C \,\}.$$

Show that F is one-to-one if and only if f is onto.